Catapult Design Lesson Plan

Purpose: This lesson consists of several related modules can be used together to develop a sense of the engineering design process or modules can be used individually. The entire series will lead students through the design of a miniature catapult using engineering design practices, including: investigating material properties, structural properties, calculating forces using Newtonian mechanics, as well as utilizing statistical and computer modeling and simulation. While the modules have been designed specifically with middle school students in mind (as many of the concepts covered are introduced in the middle school setting), it can easily be adapted for high school usage by removing much of the scaffolding and allowing the students to develop the testing and simulation procedures they will be using themselves (HS-ETS1-2), emphasizing the use of Higher-Order Thinking Skills and problem-solving.

Module One – Rubber Bands, Hooke's Law, and the Spring Constant

In this module, students will learn about Hooke's Law: that the force needed to extend a spring by some distance is proportional to its distance, i.e. $F = kx$. This will then be investigated by testing a supply of rubber bands to identify the specific spring constant (k, or stiffness) of different rubber bands (CCSS.Math.Content.7.RPA.2). This prepares the students to use those same rubber bands in their model catapult design as they will be tweaking the distance the rubber band will be stretching - and therefore the potential energy of the band, the force upon the supporting structure of the catapult, and the force applied to the launched projectile (NGSS-MS-PS3-2).

Module Two – Structural Performance Investigation

In this module, students are provided with popsicle sticks, glue, and tape, and are charged with finding the baseline strength (i.e. breaking force) of a popsicle stick, as well as investigating ways of providing greater strength than individual popsicle sticks alone are capable of (i.e. lamination & trusses). Data will be collected by breaking individual, laminated, and trussed popsicle sticks (NGSS.MS-PS3-5). In addition to collecting data, students will create a statistical model (CCSS.Math.Content.8.SP.A.2, 3, & 4) to predict how the strength will continue to change with modifications to their design: i.e. by adding more layers to a laminate, or more triangles to a truss.

Module Three – Simulation and Design

In this module, students will learn to design and use a computer model (CCSS.Math.Practice.MP2, 4, & 5) to iteratively generate data on the performance of a catapult design by systematically varying aspects of that design (NGSS-MS-ETS1-4), primarily, the length of the throw arm, the amount of stretch induced in the rubber band, and the release (stop) angle of the throw arm (NGSS-MS-PS3-2). This simulation will also be used to identify the amount of force the materials used in constructing that design will be subject to (NGSS-ETS1-1), and therefore which structures and materials from module one will provide an appropriate basis for a physical design (NGSS-ETS1-3). Further, students will graphically display the characteristics of the projectile launch within the simulation environment (NGSS-MS-PS3-1).

Module Four – Catapult Assembly

With their idealized catapult design selected from the simulation, students will then construct and test a real-world model, and compare its results to those predicted by the simulation. Certain factors are likely to have been left out of the simulation that will affect the catapult in real life – air resistance and wind, to name a few. The model can be adjusted to account for these factors as well at this point.

Materials: The miniature catapults themselves can be assembled from off-the-shelf craft supplies available at Wal-Mart and most craft stores. The software used in the lesson is available for free download from the internet.

Module One

- One or more bags of rubber bands (~ $5)
- A spring scale per group (such as those offered here for ~$5 each: http://www.teachersource.com/product/137/, or a fishing scale can make a good substitute)
- One ruler or yardstick per team (~$1 each)

Module Two

- Popsicle sticks
- Glue (Wood glue, Gorilla Glue, Elmer's, or hot glue should all work) (~$5)
- Tape (~$2 per roll)
- Breaking apparatus (there are several options, using either a spring scale and weights, or a car jack and a wooden box/scaffold)

Module Three

- The Scratch programming environment (available free at http://scratch.mit.edu)
- A compatible computer (Scratch runs on PC, Mac, and Linux computers, but currently not on the iPad)

Module Four

- One or more bags of rubber bands
- Popsicle sticks
- Glue or Tape
- Paper
- Small projectiles (corks, buttons, nuts, gummy bears, etc).
- Dixie cups (to serve as a target, optional)

Safety: Participants should wear eye protection, as splinters and projectiles can damage eye tissues. Also, some glue has strong vapors; a well-ventilated area is advised.

Background: This lesson was developed by a Fellow/Teacher partnership as part of K-State's INSIGHT GK-12 program as a way of bringing science and engineering into an industrial arts class.

It has been modified to use miniature (instead of a full-scale pumpkin chucking) catapult. It also demonstrates how data collection, statistical analysis, and computer modeling are used as part of the engineering design process.

Objectives: By working through the modules, students will accomplish the following:

Module One

- Students will systematically test the force created by stretching rubber bands to different extents (NGSS-MS-ETS1-4).
- Students will learn about Hooke's Law and be able to calculate the spring constant from the above series of distance/force measurements (CCSS.Math.Content.7.RPA.2, NGSS-MS-PS3-2).
- Students will generate a constraint - the maximum force possible from a stretched rubber band before breaking (as well as the force that will be applied to the supporting structure) – which will be used in designing their catapult (NGSS-MS-ETS-1-1).

Module Two

- Students will learn how structural arrangements interact with the properties of materials to create stronger composite building materials.
- Students will systematically test and collect data upon the ability of various structural configurations to absorb/dissipate energy before breaking (NGSS-MS-PS3-5).
- Students will create statistical model using collected data to predict how further structural changes would affect the composite structure's strength (CCSS.Math.Content.8.SP.A.2, 3, & 4).
- Students will present their findings to the class and discuss how to incorporate that knowledge into the designs of their catapults (NGSS-MS-ETS1-3).
- Students will generate a constraint – the amount of force that various structural arrangements can withstand in the design of their catapult (NGSS-MS-ETS-1-1).

Module Three

- Students design and use a computer model of their catapult design (CCSS.Math.Practice.MP2, 4, & 5).
- The model will use Newton's Third Law to predict the motion of the projectile during a launch (NGSS-MS-PS2-1).
- Using the model they iteratively generate performance data by systematically varying aspects of the design and select the best performing design for real-world implementation (MGSS-MS-ETS1-4).
- The varying aspects include the length of the throw arm, the amount (distance) of the stretch induced in the rubber band, and the release angle (NGSS-MS-PS3-2).
- Students will also work within the structural constraints identified by their earlier investigations with the rubber bands and popsicle sticks (NGSS-ETS1-3).
- Students will also add to their simulation a visual simulation of the projectile launch (NGSS-MS-PS3-1).

Module Four

- Students will select an idealized catapult design from their simulation efforts and construct a working model (MS-ETS1-3).

- Students will systematically test the physical catapult to confirm its relationship with the simulation, and identify any disparities between them (MS-ETS1-4).

Discussion questions: Catapults were originally designed as siege machines for use in warfare, and evolved alongside military tactics and fortifications. In the modern day, this role has been largely superseded, yet catapults are still built on a regular basis by re-enactors, enthusiasts, and hobbyists. Building pumpkin throwing catapults have evolved from a hobby into an entire sport with national competitions (see http://www.punkinchunkin.com/).

Module One
What is Hooke's Law?
Hooke's Law describes the force generated by a spring at different extensions. The formula is F = kx, where F is the force generated, x is the distance the spring has been stretched, and k is a constant factor known as the "spring factor". In reality, k is a simplification of the molecular forces operating within the material as it is stretched.

What does the spring scale measure?
A spring scale utilizes Hooke's law to measure force – it has a known spring constant, and the marks along the scale are at known x values, which correspond to a particular measure of force. A scientific spring scale will likely be calibrated to measure Newtons; but other measures of force (grams, ounces, and pounds) are also possibilities. In the latter cases, it may be necessary to convert the measured units to Newtons.

How can you calculate the spring constant of a rubber band?
In order to calculate k, we need to have known values for the other two terms in our expression F=kx; thus we need a force and a distance measurement. This can be obtained by anchoring one end of the rubber band (wrapping around a desk leg is a simple mechanism for accomplishing this) and attaching the other to our spring scale. The ruler should be used to measure the amount of stretch added to the rubber band as it is pulled back. Both the measured force and distance should be measured at several different stretch points. We can then plug in this data to our re-written equation k = F/x to calculate our spring constant.

Does our data reveal any surprises?
A rubber band is not a perfect spring – if we graph several data points we will see a slightly s-shaped curve emerge. This provides a rich opportunity to discuss the elastic properties of rubber and discuss possible explanations for the data results, as well as investigate a line of best fit. It also introduces important considerations in engineering, where the materials we work with may not possess "ideal" properties, but nonetheless are what we must use.

What spring constant value should we use in designing our catapult?
Different strategies can be employed here – but the two that make the most sense are to use the one produced by our line of best fit, or the one closest to our expected stretch distance.

Module Two
How can we create a structure stronger than its basic material properties? *Two answers commonly*

used in industry that work well for popsicle stick structures are laminates – layers of material glued together – and trusses – a series of triangular braces.

How can a laminate be stronger than a single piece of material of the same thickness? *Wood, from which popsicle sticks are manufactured, is harvested from trees, which have a distinct growing and resting cycle every year, tied to the seasons. The "rings" in a tree reflect this – the dark ring is the period in which the tree is not growing, while the lighter, thicker regions between rings are the cellulose remains of cell walls grown during the growing season. When wood is harvested, these rings become the "grain" of the wood. The grain is less strongly bound together than the wood between the grain – which is why wood will often splinter along the grain. Laminating multiple boards or popsicle sticks together offsets the grains in successive layers, which reduces the weakening effect of the grain.*

Why do truss structures provide greater strength than a single member? *Trusses are designed to direct forces – either tension or compression, or both – through the network of triangles, allowing the choice of material and its orientation to be arranged in a manner best suited to countering the forces the truss is subject to. However, this does mean that trusses need to be designed specifically for an application, with a clear idea of where the stress will be applied to the structure. For example, in a catapult we would want the launch arm to strike the stopping truss at the point where it is best able to absorb the force of the arm, i.e. where several triangles come together to a point, rather than in the middle of the side of a single triangle.*

Additional Resources: A number of excellent websites exist on building and breaking popsicle stick structures – most notably bridges. If you want to expand the lessons, utilizing these can be an excellent research opportunity for students, and it also requires them to re-apply knowledge from a different domain (bridge structure vs. catapult structure). One good site is this one: http://andrew.triumf.ca/andrew/popsicle-bridge/. Additionally, there are rich materials online discussing trusses, such as: http://blog.makezine.com/2010/06/10/ask-make-how-do-trusses-work/.

Module Three

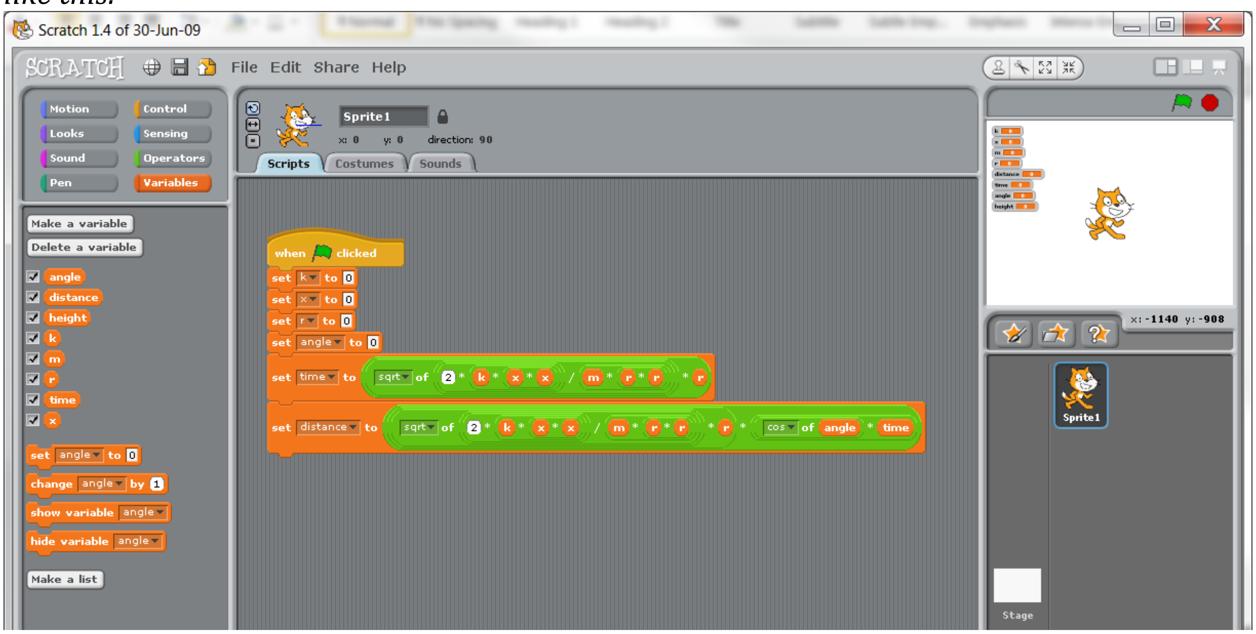
What is the purpose of a catapult? *Catapults are best-known as a category of medieval siege engines; machines built to help overcome the defenses of a castle or similar fortification by flinging rocks or other missiles at or over walls. There are many variations on catapult designs, from ballista, magonels, torsion catapults, and the trebuchet, which utilize different interplay of forces in their launching mechanism and a different trajectory. Modern catapults are used to launch jets from aircraft carriers, throw clay pigeons for trap shooting, and to set records for flinging pumpkins (a'la the television show* Punkin Chuckin *covering the World Championship Pumpkin Chunkin competition produced by the Science Channel).*

How does a catapult work? *All catapults work by converting potential energy into kinetic energy in a launched projectile. However, the manner in which the potential energy is created varies dramatically – from direct tension (i.e. stretched rubber bands, a metal or wood bow, etc), torsion (twisted ropes or bands), or gravity (dropping weights). Additionally, many catapults harness centrifugal force through a throw arm arm, sling, or combination of both to allow for longer acceleration periods and varying of launch characteristics.*

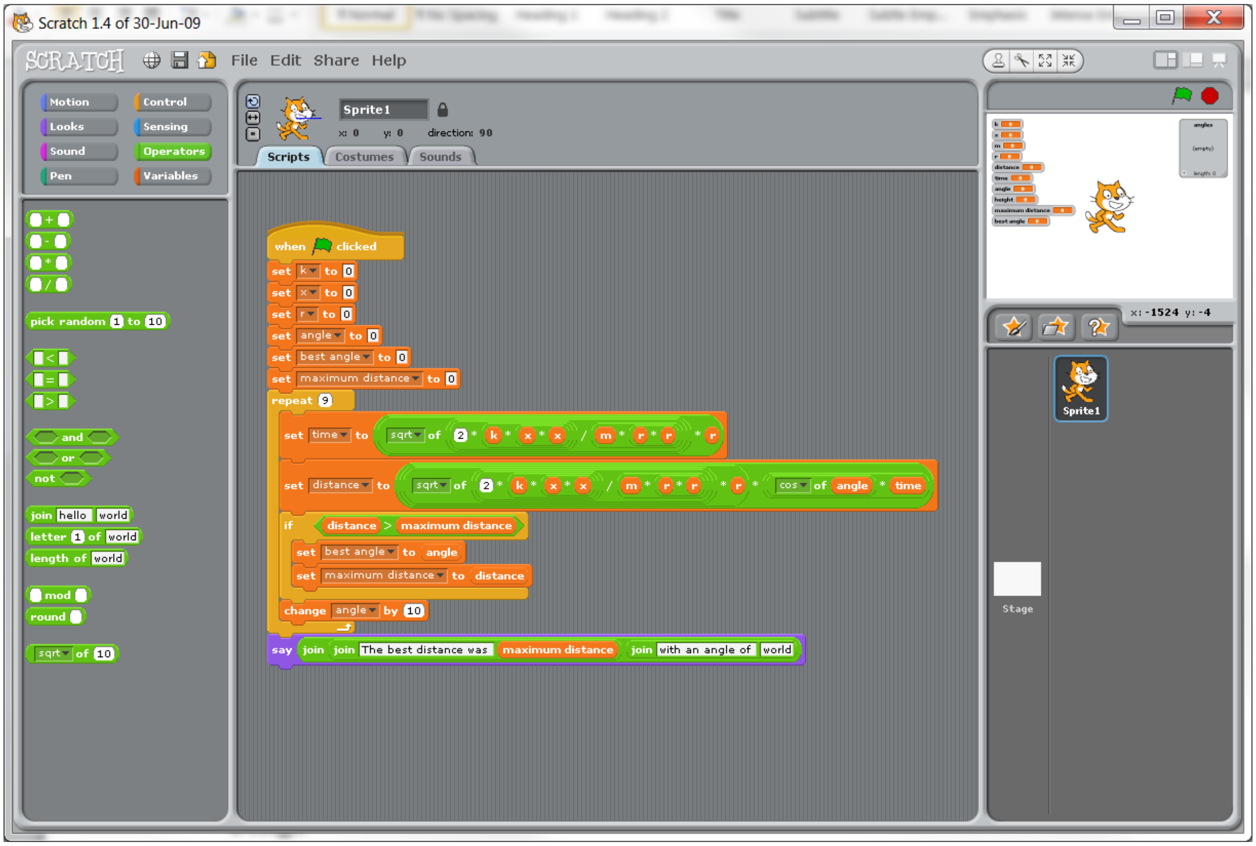
How can we mathematically represent the physics of a catapult? *Here, what we mean is "How do we express the physics of a catapult launch as an equation. This site has a thorough discussion of the physics equations involved:* https://sites.google.com/site/physicsofcatapults/home/how-a-catapult-works-the-physics.

What properties can vary in our design & mathematical model? *For example, with a mangonel catapult, we can vary the length of the throw arm, the angle of release, and the extension of the rubber band. We could also vary the mass of the projectile, though for answering design questions it is often better to hold this constant.*

How can we express our mathematical model using a programming language? *We need to create variables for each term in the equation, and write the equation itself in the programming language. For example, using the Scratch programming environment the calculations for a mangonel will look like this:*

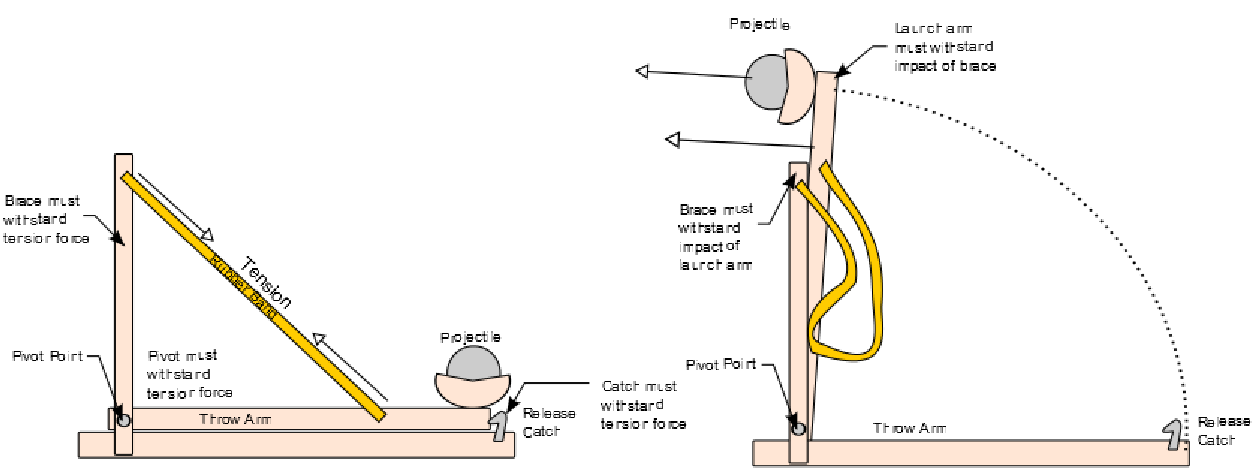


How can we systematically determine the best values for each of the design variables we have identified? *We can iteratively test every possibility using a series of loops. We'll also need to store the "best answer" to each as we move along. A simple example of finding the longest distance by varying the angle is:*



Module Four

What forces are involved in our catapult design? *In designing a catapult, we need to consider the amount of force our catapult structure must withstand. For example, most mangonel catapult designs incorporate a "stopping beam" that a catapult arm slams into to stop its progress; because the projectile is not hampered, it continues to travel forward. But the stopping arm must absorb the full force generated by the moving launch arm, else the catapult will break and require repair before re-use.*



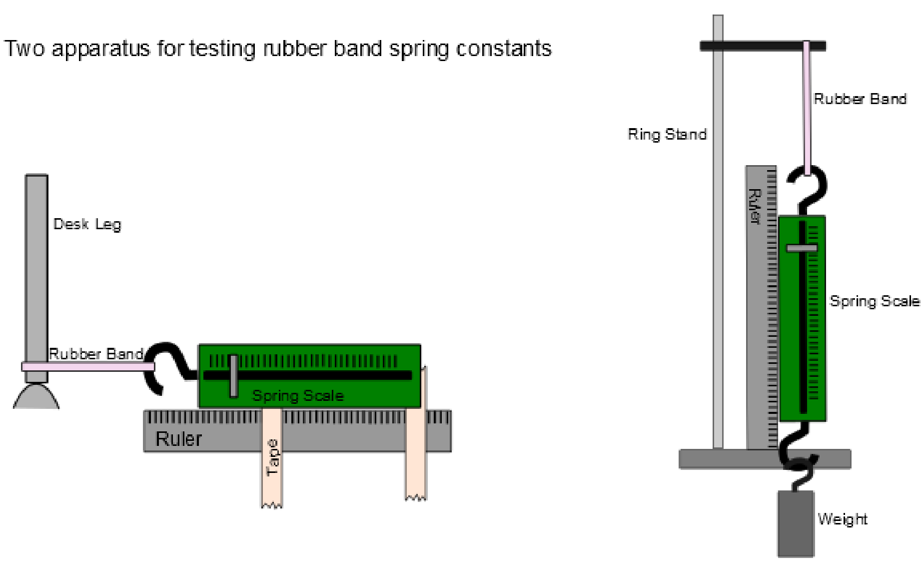
What differences do we see between our physical and computer models' performances? What explanations might exist for this disparity?
It is almost certain that you will see differences between the two models. The computer simulation ignores factors like air resistance, friction, and wind. Also, as previously mentioned, rubber bands are not an ideal spring, so the k value we are using may not be as accurate as we would like.

Procedure or Directions: The full lesson will take several days to carry out. Module one can be accomplished in a single class period. Module two is best split into two – one to construct laminate/trusses and one to break them and collect data (for a more thorough investigation, multiple building/breaking days can be incorporated, allowing students to address gaps in knowledge found after each cycle). Module three can be done in a single class period, or broken up into several, depending on how deeply you want to simulate the catapults, and if you wish to build a graphical representation. Module four will typically take two class periods (one to build the model catapult and one to test it, with drying time between).

Module One

1. Divide the class into small groups (3 is ideal). Each team receives a package of rubber bands, a ruler (or yardstick) and a spring scale.
2. Student groups will need to devise an experimental setup to stretch the rubber bands to a known distance and measure the force of the band. This can be done by anchoring one side of the rubber band to a stationary object (such as looping the rubber band around the leg of a desk with a student sitting in it) and hooking the other end of the rubber band around the hook of the spring scale. Then the students draw back the spring scale while measuring the extension with the ruler, and write down their results. A more refined approach uses a ring stand and weights – the rubber band is anchored to the ring stand, the spring scale hangs from the rubber band, and weights are attached to the spring scale to provide a constant downward force to stretch the band.

Two apparatus for testing rubber band spring constants

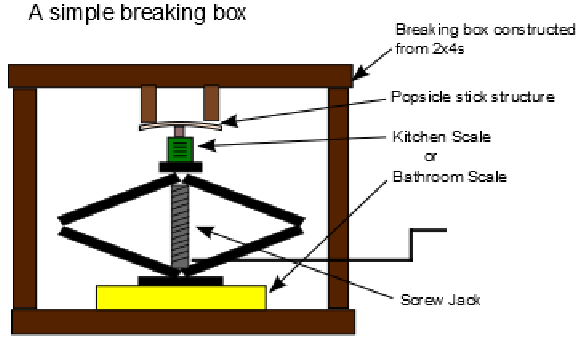


3. Once several data points (extension vs. force) have been recorded for the rubber bands, the

students calculate the values of k for each of the data points. With an ideal spring the value of k at every data point would be the same; for a rubber band this will not be the case. Reasons the data may not be ideal should be discussed at this point, at a level appropriate for the students.

4. Students will create a statistical model to represent the value of k utilizing the data points they have collected. Depending on the mathematical preparation of the class, this model could be as simple as using the average of all measured values, or it could involve using the method of least squares to find a line of best fit.
5. Students should consider what their experiments tell them about the properties of rubber bands, and how that should influence their catapult design.

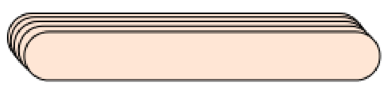
Module Two



1. Begin by demonstrating the breaking device with a single popsicle sick. A simple breaking box can be constructed from 2x4's, an automotive screw jack, and a bathroom scale (as shown in the diagram to the left).

2. As you apply increasing amounts of pressure to the popsicle stick, record the weight registered on the scale. (be sure to calibrate the scale to account for the weight of the screw jack). Continue until the popsicle stick breaks – this is your breaking point.

3. Now, have the work together to design several possibilities for a stronger structure – likely they will zero in on laminates (multiple sticks glued together into one thick structure) or trusses (sticks glued in triangular patterns), or a space frame (trusses in a 3-dimensional pattern) For younger students, you may need to provide guidance in finding appropriate forms. For older students, researching structures online may be a good starting point.



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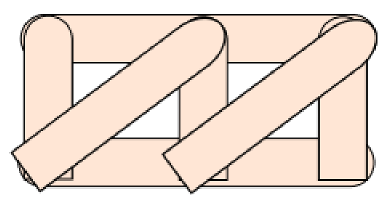

A truss

4. Once an appropriate number of designs have been devised, challenge the students to develop questions about how changes may affect their performance, i.e. will adding additional sticks to a laminate make it stronger? Does the size of the triangles in a truss affect its strength?

5. Have the students build several variants of the designs from part 3 for testing the questions raised in part 4. Remember to check the amount of time it will take the glue to dry – likely the structures will need to cure overnight or for a number of days before breaking.

6. Use the breaking box to break each of the structures the class has created, taking care to record the breaking points of each.

7. Using the data collected in part 6, have the students devise a statistical model for changes to the designs. Again, this statistical model should be appropriate for the mathematical maturity the students have achieved – i.e. average increase in strength for middle school, a regression line (line of best fit) for high school.

8. Discuss the findings and how the structure of the catapult could best be created to withstand the forces created by the rubber bands.

Module Three

1. Begin by exploring and selecting a catapult design (alternatively, you may require everyone to use the same basic design) to construct from popsicle sticks. A guide to a couple of possible designs can be found here: http://www.stormthecastle.com/catapult/the-rubber-band-powered-pyramid-catapult.htm

2. Derive or find equations for the distance and height of the catapult's projectile.

3. From those equations, identify the variables (design parameters you can change, like throwing arm length) and constants (design parameters you can't change, like the k value for a rubber band).

4. Add your variables and constants to the Scratch programming environment, and then compose the calculations for calculating the projectile distance and height.

5. Calculate a few possibilities by manually changing variable values and running your calculations.

6. Add looping to iterate over a set of possible values for a single variable, using Scratch to remember the best values.

7. Add nested loops to loop over possibilities for the remaining variables, using Scratch to remember the best calculated values and the variable values that achieved them.

8. Add the calculations for the x and y position of the projectile in flight, and use them to draw the position of the projectile (or move a Scratch sprite to that position) at one-second intervals for the duration of the flight (which is calculated in the first equations).

Module Four

1. Design the structure of your catapult using the structures evaluated in Module 2, to accommodate the forces generated by your projectile solution. (You may also use Scratch to

calculate these forces as well).

2. Construct your catapult design using the supplied materials (popsicle sticks, rubber bands, glue or tape).

3. Systematically test your design – i.e. run multiple tests – and compare your results to those predicted by your computer model.

4. Identify where the predictions and collected data differ, and provide an explanation for why this might be.

5. Tweak your computer model based on your reasoning.

Extension or Exploration: Rather than focusing solely on distance, you might consider the force generated on impact between the projectile and the target – remember the goal for many early catapults was to maximize the damage to fortifications. Also, you could consider incorporating the engineering of fortifications into your lessons; siege engines and fortifications evolved hand-in-hand, and you could have your students explore this relationship in a very hands-on manner.

Alternatively, you could look at modern catapults, like those used to set world records for pumpkin flinging (a great resource for this is http://www.punkinchunkin.com). Additional things to consider when flinging a pumpkin: the projectile itself becomes an important factor. Pieing (where the pumpkin explodes as it is released from the catapult) requires students to consider what the maximum acceleration a pumpkin can withstand, and modifications are required to catapult designs to accommodate this (also, another module can be added to determine the “maximum” acceleration the average pumpkin can tolerate before pieing). Students could also build full-scale pumpkin catapults utilizing the lessons they have learned through their model efforts.